GEOMETRY FOR THE ARTIST: AN INTERDISCIPLINARY CONSCIOUSNESS-BASED COURSE

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Abstract. Geometry is an inherent aspect of any work of art. Both geometry and art are fundamentally products of the consciousness of individuals—the mathematicians who create geometry and the artists who create art. We therefore expect geometry and art to embody qualities of consciousness. This understanding is the basis for *Geometry for the Artist*, a mathematics course at Maharishi University of Management that explores how certain topics in geometry (symmetry, perspective, fractals, non-Euclidean geometries, and topology) are connected to art, and, moreover, how the understanding of consciousness developed by Maharishi Mahesh Yogi helps us see connections between art and geometry. This paper describes specific mathematical topics studied in this course, how they are used in art, and their relationships to consciousness.

1. Introduction

Art is subjective, depending on the emotions, intentions, life experiences, culture, training, and skill of the artist. As Kasimir Malevich (1878–1935) puts it, "Every work of art—every picture—is the reproduction, so to speak, of a subjective state of mind—the representation of a phenomenon seen through a subjective prism (the prism of the brain)" [38, p. 40]. The appreciation of art is likewise subjective, depending on subjective characteristics of the viewer.

Studio training and the study of art history can culture the ability of the artist to create art and the ability of the viewer to appreciate art. In addition, the intellectual approach of mathematics can be valuable to the artist and the viewer. According to Tony Nader:

> Certain forms of art also appeal to the intellect. The intricate symmetries and constructs in architecture and in classical music, for example, sometimes entice an intellectual analysis that helps reveal their beauty. As knowledge has organizing power, intellectual understanding of certain aspects of art awakens a greater appreciation for them. [43]

The premise of the course *Geometry for the Artist: From Point to Infinity* at Maharishi University of Management is that the study of geometry in relation to works of art promotes an intellectual understanding that can lead to greater appreciation of both geometry and art. To that end, the course covers several major topics in geometry, how artists use them, and how they are connected to consciousness. Students have found this approach valuable. They have said:

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- With a course like this, learning how unbounded consciousness is the source of all art expression and geometry has open my eyes to boundless potential for my own self-expression.
- Very well done in this course of showing the infinite value in shapes and how artist and mathematicians are very much the same in their expression. I like how we always related the lessons to Maharishi Vedic Science. I have a totally different view toward math thanks to your class.
- This class and the context it was taught in actually facilitated a connection and relationship to the knowledge.
- Consciousness gives life to knowledge and a sense of purpose to daily activities, including school and homework.

This paper will describe the course and show the value of this interdisciplinary approach.

2. Overview

To justify the premise of the course, namely the importance of geometry for the study of art, the next section, Section 3, will present evidence that artists use geometry in their work in a significant way.

Section 4 introduces Maharishi Science and Technology of Consciousness, the scientific approach to the study of consciousness used in the course, and describes its value for the students.

The next sections introduce five topics of geometry (symmetry, perspective, fractals, non-Euclidean geometry, and topology), how they appear in art, and the insights given by Maharishi Science and Technology of Consciousness.

Symmetry in art has properties of balance and harmony. The mathematical interpretation of symmetry in Section 5 demonstrates the qualities of silence and dynamism belonging to consciousness.

Perspective, discussed in Section 6, gives mathematical procedures for creating a picture that shows a scene just as the artist saw it. The description of the structure of knowledge given by Maharishi Science and Technology of Consciousness perfectly describes the situation of a perspective picture: knowledge (of the scene) is the coming together of knower (viewer), process of knowing (the picture), and the object of knowledge (the scene).

Fractals are present everywhere in nature and their representations are therefore present in art; see Section 7. Fractals are generally created by a process of repetition—or self-referral—and can appear in the work of an artist as the result of the artist's self-referral creative process.

Non-Euclidean geometries, described in Section 8, give an understanding of the properties of surfaces of natural objects. At the same time, by extending the familiar geometry of Euclid, non-Euclidean geometries show that there exists a range of possibilities for geometry, similar to the range of possibilities of consciousness.

Topology, the final area of geometry given in Section 9, is the subtlest of all the topics studied in the course. Topology extends the range of geometry from the concrete level of Euclidean geometry to a very abstract level.

Finally, we consider some themes from the study of consciousness more broadly and how they are connected to geometry and art and conclude with some students' reflections describing what they have learned from connecting geometry, art, and consciousness.

3. Do artists really use geometry?

When looking at a work of art that appears to be a beautiful and complete subjective expression of the artist, one might wonder whether the artist intentionally used the mathematical discipline of geometry in the creation of the work. One might be concerned that intellectual analysis using geometry would find something that the artist did not really intend or plan.

Our discussion below will show that artists make explicit and implicit use of geometry and will support our approach that using geometry is fruitful in the analysis of a work of art.

3.1. **Explicit use of geometry.** During the Renaissance, the influence of mathematics on artists was substantial. New appreciation for Euclidean geometry led to the modern theory and techniques of perspective, which were introduced in the early fifteenth century by Italian artists Filippo Brunelleschi (1377–1446) and Leon Battista Alberti (1404–72).

Alberti devoted a large part of his treatise *Della Pittura (On Painting)* to the application of mathematics to painting. This included a significant new development in the mathematics of perspective—the technique for constructing a checkerboard or grid in perspective—shown in Figure 1. Alberti concludes his discussion of painting with recognition of the importance of geometry for the artist, saying, "Therefore, I believe that painters should study the art of geometry" [5, p. 88].

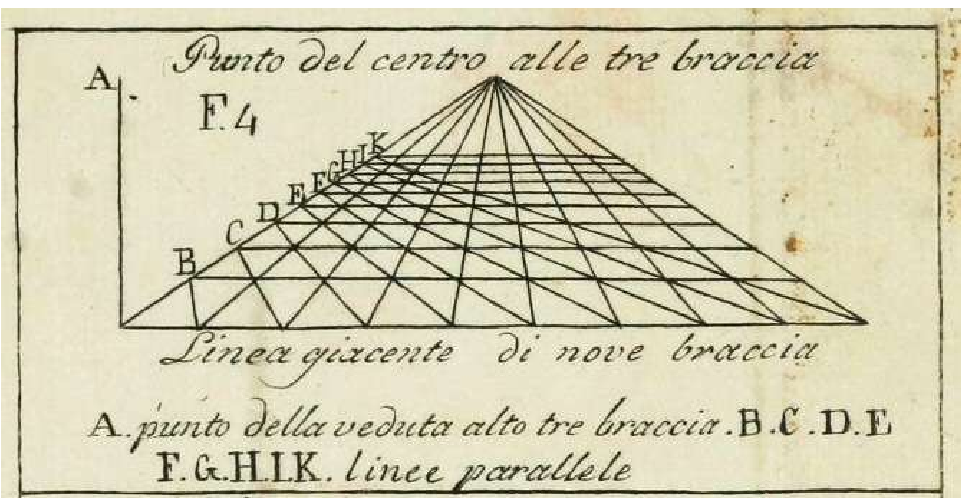

Figure 1. Diagram by Leon Battista Alberti of perspective lines leading to a vanishing point from his treatise *Della Pittura*

Other Renaissance painters who used the geometry of perspective in their work were Paulo Uccello (c. 1397–1475), noted for his dramatic and forceful use of perspective; Piero della Francesca (c. 1415–92), a mathematician and artist, who wrote the treatise on perspective *De Prospectiva Pingendi (On the Perspective of Painting)*; and Leonardo Da Vinci (1452–1519), a master of geometric techniques of perspective, as shown in Figure 2.

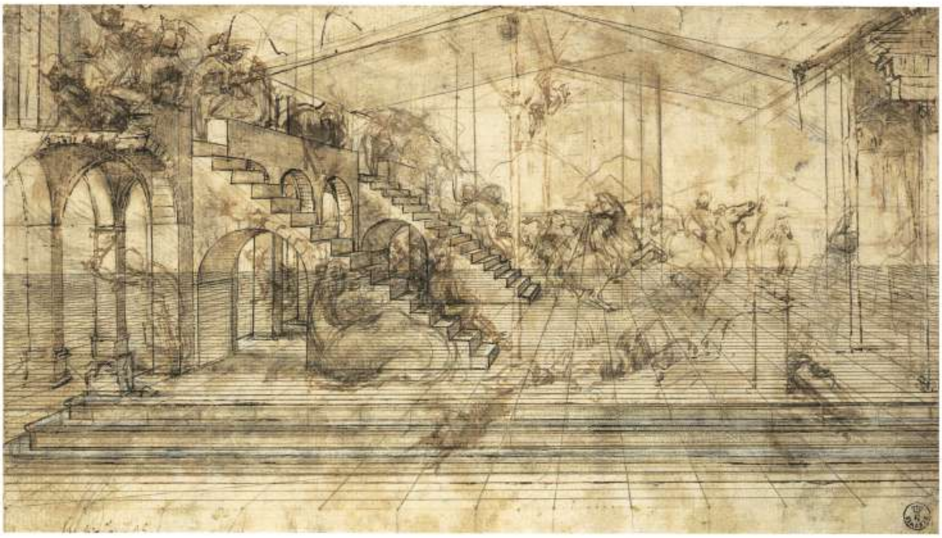

FIGURE 2. Leonardo da Vinci, Perspective study for the background of the *Adoration of the Magi*, 1481 (Dover)

FIGURE 3. Albrecht Dürer, *A man drawing a can*, 1538 (Dover)

The German artist Albrecht Dürer (1471–1528) learned about perspective and other uses of geometry during his trips to Italy in 1495 and 1505. He later consolidated this knowledge in *Four Books on Measurement* and *Four Books on Human Proportion*, the first works in German to describe the mathematical basis of art.

The care with which he presents the theory of perspective is suggested by the woodcut shown in Figure 3. This picture from *The Art of Measurement* illustrates the concepts of station point (shown as the hook on the wall at right), visual ray (the string and tube held by the artist), and picture plane (the blank pane of glass on which the artist is drawing).

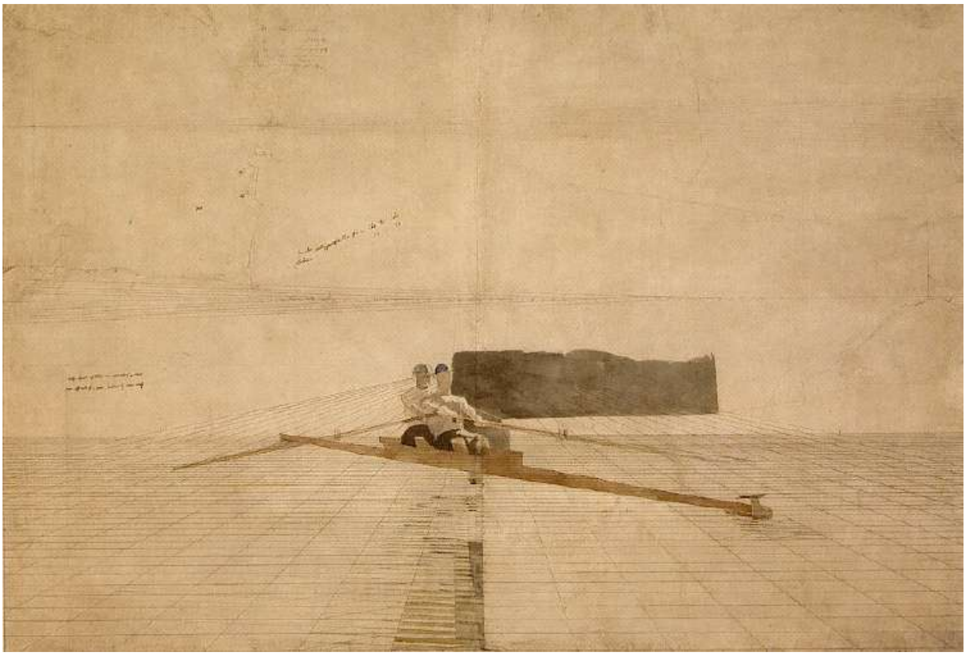

FIGURE 4. Thomas Eakins, *Perspective Drawing for "The Pair-Oared Shell,"* 1872

The American artist Thomas Eakins (1844–1916) made extensive use of perspective, creating carefully detailed perspective studies such as the one shown in Figure 4. His textbook *A Drawing Manual* [19] deals almost exclusively with perspective.

There are many examples of geometry in art other than the use of perspective. Kasimir Malevich developed a style he called *Suprematism*, which "used only geometric shapes and a limited colour range" [11]. One example is his painting *Black Square*, a black square on a white ground.

Several schools of art in the late nineteenth and early twentieth century adopted geometric themes: cubism, analytic cubism, synthetic cubism, geometric abstraction, and pointillism. Artists sometimes indicate the influence of geometry on their work in the title—*Black Circle* and *Suprematism Painting: Eight Red Rectangles* by Malevich as well as *Squares with Concentric Circles*, *Circles in a Circle*, and *Several Circles* by Wassily Kandinsky (1866–1944).

Salvador Dalí (1904–89) had a deep and lasting interest in mathematics, working closely with René Thom (1923–2002), the French topologist who introduced him to catastrophe theory [40]. Dalí's interest in topology is shown in the topologically transformed clocks of *Persistence of Memory* (1931). Thomas Banchoff (b. 1938), American geometer, helped Dalí understand higher dimensions of space [9]. The cross in Dalí's *The Crucifixion—Corpus Hypercubus* (1954) can be interpreted as the unfolding of a four-dimensional hypercube.

The unexpected and riveting work of M.C. Escher (1898–1972) was influenced by the knowledge of mathematics that he acquired from many different sources. A 1922 paper by the Hungarian mathematician George Pólya (1887–1985) [50, 51] inspired many of Escher's symmetry drawings. A picture sent to him of the Poincaré disc by geometer H.S.M. Coxeter (1907–2003) became the basis for Escher's *Circle Limit* series [51]. The Penrose tribar, an impossible figure devised by British mathematician and physicist Roger Penrose, motived pictures such as *Waterfall* and *Ascending and Descending* [21, 51].

Today, there is great sharing of ideas and techniques among artists and mathematicians. The Bridges Organization has yearly conferences [1] that bring artists and mathematicians together to "bridge" their areas of interest. The American Mathematical Society includes the yearly Mathematical Art Exhibition [2] in their annual meeting. The Mathematical Association of America has a Special Interest Group on Mathematics and the Arts.[1] Mathematics Awareness Month for 2003 [3], sponsored by the Joint Policy Board for Mathematics, was devoted to mathematics and art. *The Journal of Mathematics and the Arts*[2] was founded in 2007.

3.2. **Implicit uses of geometry in art.** We now consider how geometry appears in art in less explicit ways.

First, note that the forms and structures in nature that inspire artists are the same as those that motivate the development of geometric ideas. For example, symmetry appears over and over in nature: the bilateral symmetry of the human body; the radial symmetry of the sun; and the three-dimensional symmetry of crystals. Most flowers have symmetry, ranging from the bilateral symmetry of an orchid to the three-fold symmetry of a tulip, to the five-fold symmetry of an apple blossom, to the spiral symmetry of the sunflower. Many animals—mammals, birds, reptiles, and insects—have bilateral symmetry. Starfish and other marine life have five-fold, and sometimes seven-fold, ten-fold, or even fifty-fold, symmetry [28].

Artists inspired by such symmetrical structures in nature will of course incorporate symmetry into their work. We see this, for example, in full-face portraits and masks and sometimes in flower or animal paintings. Naturally occurring symmetrical designs are often the inspiration for symmetrical designs, borders, and tilings, as seen in the medieval border in Figure 5 that is based on a flower motif.

The mathematical study of symmetric patterns was motivated in part by the need of crystallographers to describe the symmetry of crystals [52, p. 16ff].

Geometric laws govern how light travels and how we see the shapes around us. The laws of perspective, based on these geometric laws, determine how three-dimensional shapes appear to the eye. Thus, an artist obeys the geometry of perspective when painting what is seen onto a two-dimensional canvas, even though not explicitly using mathematical rules. For rectilinear shapes in the man-made

[1] `http://sigmaa.maa.org/arts/`

[2] `http://www.tandfonline.com/toc/tmaa20/current`

Figure 5. A Medieval Border, France

environment, of course, these geometric laws are very concrete and striking, but these laws also apply when looking at and painting a natural scene.

As Maharishi points out [24, p. 41], "Art is the expression of life; it is the expression of creation." Geometry is found everywhere in nature, so when an artist draws whatever shapes are found in nature, the images will naturally have the geometry belonging to those natural objects—the symmetry of flowers and crystals, the fractal geometry of trees and mountains, and the non-Euclidean geometry of the human body.

3.3. **Using geometry to analyze art.** We have seen that artists use many types of geometry in their work and paint a variety of natural geometric forms. Thus, it is necessary to use a range of geometric concepts in the analysis of specific works of art. For example, symmetry transformations are convenient for the mathematical analysis of designs, borders, and tilings. Symmetry classifications allow us to compare the symmetry of different patterns.

An understanding of perspective is necessary for analyzing a picture that was created using perspective. Geometrical methods can determine the station point (the location of the artist or viewer) and, in many cases, the viewing distance (the distance of the artist or viewer from the canvas).

Identifying congruent or similar shapes in a painting gives insight into what shapes the artist intended to connect or relate to one another. Locating the presence of fractals, curved lines and surfaces, or distortions through topological transformations gives an appreciation for how the artist viewed, interpreted, and expressed the subject matter.

Analysis of pictorial composition requires looking at a picture in terms of lines and shapes, the arrangement of shapes, and the balance created by those shapes. Such analysis deals mainly with the overall geometric structure inherent in the work [49]. For example, the dominant shapes in a painting may be arranged in a triangular or circular shape. The artist may use lines to emphasize different qualities: horizontal lines like the horizon between sky and sea give a feeling of expansion; vertical lines like trees or columns give stability; and diagonal lines like a plane taking off convey dynamism.

While the geometrical analysis of a work of art does not completely capture the full value of a work of art, it can give unique insight not available in any other way. Geometrical analysis can support and confirm conclusions from other types of analysis, and it gives a richer appreciation of the skill of the artist.

With this appreciation of the usefulness of geometry for the artist, we now turn to discussing various aspects of the Consciousness-Based course *Geometry for the Artist*.

4. The Value of Maharishi Science and Technology of Consciousness for the Student

Along with gaining objective knowledge of a discipline such as art or geometry, the student should be developing subjective qualities such as intelligence, focus, and creativity. To accomplish this, Maharishi University of Management integrates Maharishi Science and Technology of Consciousness into the structure of each academic course.

Maharishi Science and Technology of Consciousness, like any science, has two components, practical or experiential and theoretical [37, p. 271]. The Transcendental Meditation and TM-Sidhi program provides the experiential element of this science of consciousness. The Transcendental Meditation technique is a simple, natural technique practiced for twenty minutes twice daily. During the practice of the Transcendental Meditation technique, the mind settles down to its least active state, and the meditator gains the subjective experience of wakefulness without active thinking, a state of silence without activity, a state of pure awareness or pure consciousness. The TM-Sidhi program cultivates the mind to think and act from that least excited state of awareness.

The theoretical component of this science, developed by Maharishi Mahesh Yogi over a fifty-year period, is based on three sources: the wisdom contained in the ancient Veda and Vedic literature (see for example [44, pp. 1, 35 ff.]), intellectual analysis of personal experiences of higher states consciousness [46], and scientific research on the development of higher states of consciousness [4, 13, 14, 15, 45].

Maharishi Science and Technology of Consciousness is incorporated into the curriculum at Maharishi University of Management in two ways. First, students practice the Transcendental Meditation and TM-Sidhi program as part of their academic program. Second, students gain a holistic understanding of all disciplinary content by connecting the principles of each discipline they study with the principles of personal development that Maharishi has advanced; see for example [18, 33, 35].

Students gain many benefits from their regular meditation. During the practice of the Transcendental Meditation technique, the active mind, which is ordinarily processing thoughts and sensory impressions, becomes progressively less active until it transcends, or goes beyond, thoughts and sensory input and experiences its own nature, pure unbounded awareness. With regular practice, students become familiar with their own consciousness, the subtlest level of life. This leads to refinement and expansion of the awareness of the meditator outside of meditation, so that students gain a deep, personal connection with the qualities of consciousness that artists draw upon to structure a work of art. They become viewers capable of experiencing the full value of a work of art, from the level of the colors and shapes on the canvas to the finest value of emotion that the artist has embedded in those colors and shapes [24, p. 33].

Scientific research [4, 13, 14, 15, 45] has documented the enhancement of many characteristics important for students and relevant to the appreciation of art. Intelligence grows with the regular practice of the Transcendental Meditation technique [6, 17, 54] as does creativity [30]. Improved efficiency of visual perception and increased freedom from habitual patterns of perception and increased perceptual flexibility [16] give students a fresh approach when looking at a work of art. Greater aesthetic orientation [47] leads to greater appreciation of art. Research also shows that college art students develop broader comprehension and improved ability to focus attention as well as greater field independence with regular practice of the Transcendental Meditation program [25, 26, 57].

5. Symmetry

All cultures use symmetric designs for their symbols, flags, and crests. Artisans everywhere decorate pottery, fabric, and buildings with symmetric designs. Symmetry is associated with qualities of balance, harmony, orderliness, and coherence. It should not be surprising that symmetry is also a quality of the field of pure consciousness, as we shall see below.

5.1. **Symmetry transformations and their classifications.** Mathematicians use symmetry transformations to measure the degree of symmetry belonging to a mathematical structure or physical object.

A *symmetry transformation* is a motion of a shape or design that leaves the shape or design apparently unchanged. To illustrate this, consider the designs shown in Figure 6. The first design would look the same if it were flipped across a vertical line through its center as would the second. Both of these designs are said to have *bilateral symmetry*, a very familiar type of symmetry. The third and fourth designs would also look the same if they were flipped across vertical lines through their centers, but in addition would look the same flipped across horizontal and diagonal lines through their centers. These designs have *four-fold reflection symmetry.* In addition, both of these designs have four-fold rotational symmetry, which means they look the same when rotated through angles of 90°, 180°, 270°, or 360°.

Figure 6. Four symmetric designs. The two on the left have bilateral symmetry and the two on the right have four-fold reflection symmetry.

Moving a shape in such a way that it looks the same captures the intuitive idea of symmetry. If we reflect a shape across a line and it looks the same, we have shown that the two halves of the shape on opposite sides of the line look identical. Similarly, if we rotate a shape 90° about its center point and it looks the same, we have captured how four different parts of the shape look identical.

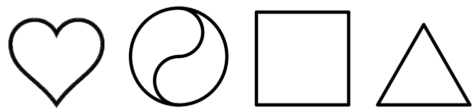

Figure 7. Four simple symmetric geometric designs

To determine the symmetry transformations belonging to a shape, we imagine that it is being moved. The first shape in Figure 7, the heart, is symmetric because it would look the same if it were reflected across a vertical line through its middle; each half is a mirror image of the other half, as shown in Figure 8. The second shape, the yin-yang symbol, is symmetric because it would look the same if it were rotated 180° about its center point. The square and the triangle have both mirror and rotational symmetry. The square would look the same if it were reflected across vertical, horizontal, or diagonal lines or if it were rotated through angles of 90°, 180°, 270°, or 360°. The triangle would look the same if reflected across vertical or diagonal lines or rotated through angles of 120°, 240°, or 360°. The dashed lines in Figure 8 show the mirror lines of the three shapes.

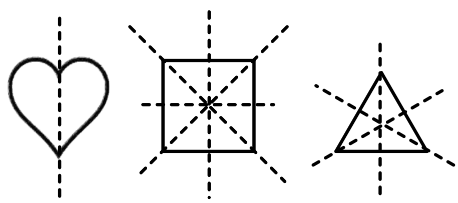

Figure 8. Three symmetric designs along with their mirror lines

The connection of the mathematical characterization of symmetry with consciousness is illustrated by verse 18 of Chapter 4 of the Bhagavad-Gita, where Krishna explains the relationship of action and inaction. The translation of this verse by Maharishi Mahesh Yogi [37, p. 278] is:

> *He who in action sees inaction and in inaction sees action is wise among men. He is united, he has accomplished all action.*

From the perspective of this verse, a symmetry transformation, such as a reflection or rotation, is an action imposed on an otherwise inactive mathematical object, so the transformation "sees action in inaction." An essential characteristic of a symmetry transformation is that the object looks the same after the symmetry transformation as it did before—the viewer cannot tell the difference between the object before the transformation and the object after the transformation. Thus, a symmetry transformation "sees inaction" of the design "in action," under the movement of the symmetry transformation.

By inaction, Maharishi means the state of Being, Transcendental Consciousness, that state of pure awareness that can be experienced through the practice of the Transcendental Meditation technique. One who sees "inaction in action" is one who experiences this state of pure awareness along with the ordinary activity of daily life. One who sees "action in inaction" experiences all activity in terms of the silence or inaction of the deepest, non-active level of the Self, the state of Being. Such an individual is fully realized and has gained the highest level of consciousness.

An individual who experiences silence along with activity and sees all activity as an expression of silence is, in Maharishi's translation, "united, he has accomplished all action." This means such an individual has attained perfection and gained fulfillment [37, p. 280]. Action is a way of fulfilling one's desire. To have "accomplished all action" means to have attained all possible goals in life, indicating that one has gained fulfillment. For the mathematician, gaining complete knowledge of the symmetry of some mathematical structure is very powerful and fulfilling.

5.2. **The beauty of symmetry.** Something that has symmetry exhibits qualities of silence or inaction along with qualities of dynamism or action. A basic shape that is not changed—silence—is repeated over and over in different positions or orientations—dynamism.

Symmetry is beautiful and fascinating; it is found everywhere in nature; and it is a prevalent theme in art, architecture, and design in cultures all over the world and throughout human history. From the charm of a snowflake to the deep spirituality of Leonardo's *Last Supper*, symmetry has an essential role in nature and art.

Pure consciousness has remarkable qualities of symmetry. Pure consciousness is unbounded and everywhere the same. Every "part" of pure consciousness looks like every other "part." Any movement of pure consciousness leaves it unchanged, so every movement of pure consciousness is a symmetry transformation. Thus, the collection of symmetry transformations of pure consciousness includes all transformations. For this reason, it makes sense to say that pure consciousness has the greatest possible symmetry.

This analysis concurs with Maharishi's description of the importance of maintaining symmetry in physical systems. He points out the quality of symmetry that belongs to consciousness [34, pp. 181–2]:

> Maintenance of symmetry also applies to consciousness: pure consciousness, self-referral consciousness, unbounded awareness, is the most expanded, smoothest state, the one with the most expanded boundaries—it has the greatest degree of symmetry.

To see why symmetry is so attractive and aesthetically pleasing to us, in art as well as in science, consider the field of pure consciousness. According to Maharishi [37, p. 282], the silent level of life, pure consciousness, the source of thought, is subjectively experienced as bliss; whenever the active level of the mind begins to move in the direction of the silent level of the mind, it experiences increasing bliss.

The repetition of parts of a symmetrical design indicates an underlying pattern or unifying value for something that is physical and concrete. When observing a symmetric object, whether an artistic design or a mathematical structure, the mind is spontaneously led to experience the surface value of the object (activity) and the more unifying symmetric values of the object (silence) simultaneously. The charm of symmetry for the viewer is a result of this evolutionary experience of perceiving the diversity of the surface level and the unity of the silent level simultaneously.

For the scientist, the symmetry of a system is a very deep level of the organizing power that structures an object or physical system. Indeed, the most important laws of physics are those that encode the symmetry of a system [53, 58].

5.3. **M.C. Escher and Symmetry.** M.C. Escher is an artist who has used symmetry effectively. He began his intensive work with symmetry after visiting the Alhambra in Spain, where the walls, ceilings, and floors were covered with symmetric tiling patterns. Escher's purpose in creating symmetric patterns was for capturing something deeper and more powerful, for "expressing unboundedness in an enclosed plane that is bound by specific dimensions, while retaining the characteristic and fascinating rhythm" [22, p. 84]. The wide appeal of Escher's symmetric work confirms his success in this undertaking.

6. Perspective

The goal of perspective is to represent a three-dimensional scene on a two-dimensional canvas that gives the viewer the impression of viewing the three-dimensional scene rather than the two-dimensional painting. Geometric techniques of perspective make it easy and straightforward to make rectilinear shapes such as buildings, roads, fences, furniture, and tilings on a canvas appear three-dimensional.

Geometric analysis of the perspective in a picture tells us where the eye of the artist was located with respect to the canvas when the picture was painted, which is where the viewer should be located to see the picture as the artist intended. Knowing the location of the viewer—whether looking from above, below, near, far, straight on, or at an angle—indicates how the artist is connecting the viewer to the scene. The viewer might feel to be an intimate part of the activity in the picture, overwhelmed by the drama or significance of the events, or as if a dispassionate bystander. In many medieval religious works, for example, the viewer is uninvolved. Renaissance paintings make viewers feel as though they are intimate and involved. Impressionist paintings, on the other hand, don't give a clear-cut location for the viewers but take the viewer into the mind and heart of the artist.

The woodcut *An artist drawing a seated man on to a pane of glass through a sight-vane* by Albrecht Dürer, shown in Figure 9, is an example of a picture that makes us feel as though we are an intimate part of the activity. This picture is drawn using perspective while also showing the elements of constructing a perspective picture: the station point where the eye of the artist is located, the picture plane on which he is drawing, and the scene he is painting. Analysis of the perspective tells us that the viewer of this woodcut is in the room, close to the scene, at eye level with the standing artist, between the artist and the seated man, observing the activity of the artist in a familiar, personal way.

Many artists do not adhere rigidly to the rules of perspective, manipulating them to achieve a specific result. For example, in *The Resurrection*,[3] Piero della Francesca lets the viewer look directly into the face of Christ even though the viewer is situated below the soldiers guarding Christ's tomb at the base of the picture. Pablo Picasso (1881–1973), in paintings such as *Girl before a Mirror*,[4] shows us the subject from

[3] `https://commons.wikimedia.org/wiki/File:Resurrection_(Piero_della_Francesca).jpeg`
[4] `https://www.moma.org/collection/works/78311`

FIGURE 9. Albrecht Dürer, *An artist drawing a seated man on to a pane of glass through a sight-vane*, 1525, woodcut (Dover)

more than one point of view. M.C. Escher bends the rules of perspective just enough to deceive the viewer, as described below in Section 6.2. Applying the rules of perspective helps the viewer understand the intention of the artist in pictures such as these.

6.1. **Knower, known, and process of knowing.** Maharishi Science and Technology of Consciousness helps us understand the relationship of artist, viewer, painting, and scene, which are so significant in perspective pictures.

Looking at a picture is a process of gaining knowledge. Maharishi maintains that any experience of knowledge has three components:

> Knowledge naturally involves three things: the knower, the object of knowledge, and the process that connects the knower and the object—the process of knowing. [24, p. 90]

The value of a work of art depends on the quality of all three components. Of these three, Maharishi identifies the component of the knower as key:

> However, without having established the "I"—the subjective aspect of knowledge—the object will not be fully located. In the field of knowledge, it is necessary that the knower be established first, and on the basis of establishing the knower, the object is known. From this analysis, we can see that very naturally the knower is the first point of reference in knowledge, the object of knowing is the second point of reference, and the process of knowing that connects the two is the third point of reference. [24, pp. 90–91]

For this reason, the development of the subjective aspect of the knower—the viewer—is essential for full appreciation of any work of art. This is particularly true for a perspective painting since the viewer has such an essential role. And, as we have seen in Section 4, students' regular practice of the Transcendental Meditation technique enhances the development of inner subjective qualities and perceptual abilities.

6.2. **M.C. Escher and the manipulation of perspective.** Escher's woodcuts from his earlier years in Italy demonstrate that he was a master of perspective, carefully positioning the viewer with respect to a three-dimensional scene to create a specific effect. In the woodcut *Tower of Babel*[5] from 1928, Escher used three-point perspective to locate the viewer high above the construction activity of the tower, giving a feeling of the height and grandeur of the tower. About *Cubic Space Division*[6] and *Depth*,[7] Escher said, "My only intention was to suggest an impression of three-dimensionality, of endless depth" [22, p. 56] and he succeeded admirably in this.

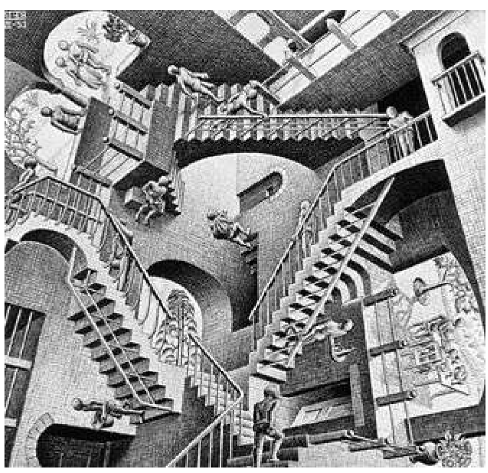

FIGURE 10. M.C. Escher, *Relativity*, 1953, lithograph

[5] http://www.mcescher.com/gallery/italian-period/tower-of-babel/

[6] https://www.nga.gov/Collection/art-object-page.54254.html

[7] http://www.mcescher.com/gallery/recognition-success/depth/

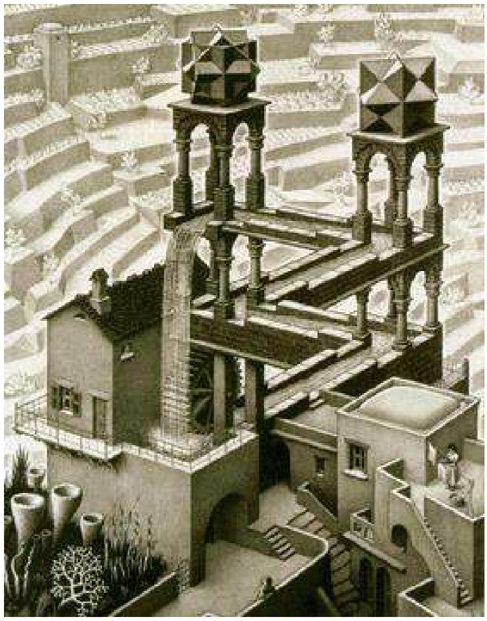

Figure 11. M.C. Escher, *Waterfall*, 1961, lithograph

In his later works, Escher carefully used variations of linear perspective to create images representing impossible realities. To understand Escher's intention and to gain full appreciation of his skill, the viewer must understand the mathematical properties of perspective. The lithograph *Relativity*, which uses perspective correctly, shows sixteen people walking, sitting, and climbing stairs; see Figure 10. When each person or group is viewed individually, everything looks fine. But when the picture is looked at as a whole, we see that the alignment changes from group to group. The zenith of one group is the nadir of a second group, the right-hand vanishing point of a third group, and the left-hand vanishing point of yet a fourth group. These inconsistencies completely confuse the viewer, as Escher intended.

In pictures like *Waterfall*, Figure 11, Escher uses the principle that an object farther away from the picture plane appears higher in the picture. The viewer should interpret the zigzag of water that travels higher in the picture as water that is moving farther away; however, Escher connects the highest point of this zigzag of water to the top of the waterfall, which appears to be close. This makes the viewer interpret the water to be going up rather than away and Escher's mastery of perspective again confuses the viewer.

7. Fractals

Benoit Mandelbrot (1924–2010), one of the founders of fractal geometry, asserted that the geometry of Euclid was not the most effective way to study shapes seen in nature, but rather it was fractal geometry that could best model nature. He referred to this geometry as the "geometry of nature" [39, p. 1] because natural shapes such as clouds, mountains, waves, coastlines, rivers, ferns, and trees are fractals. The fractal shape of lightning is shown in Figure 12.

FIGURE 12. Lightning.

To describe the structure of fractals, we need the concept of similarity: two shapes are *similar* if they have the same shape but possibly different sizes, as shown in Figure 13. Scaling one of the two similar shapes up or down can make it look exactly like the other.

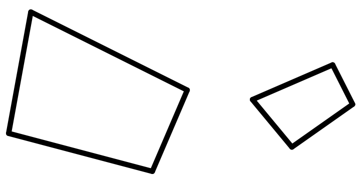

FIGURE 13. Two similar shapes

A fractal is a geometric shape that is similar to itself; this is the property of *self-similarity.* This means that a fractal looks like itself when scaled up or down. This is shown in the Koch snowflake curve, Figure 14, where the top part of the curve is similar to the whole curve; the top part looks like the whole curve when scaled up.

7.1. **The self-referral structure of fractals.** The self-referral construction of a fractal is an example of the self-referral dynamics of consciousness described by Maharishi [35, pp. 10–11]. The silent value of pure consciousness interacting with itself is the fundamental self-referral process of creation [35, p. 185]. Maharishi sees the world around us as the result of this self-referral dynamics of the field of intelligence, which is the same unified field of natural law recognized by physicists:

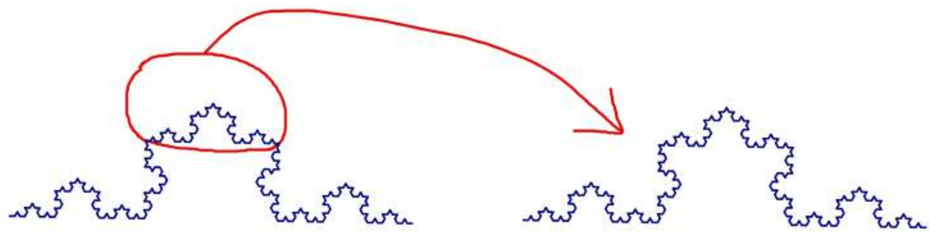

Figure 14. When a small part of the Koch snowflake curve is scaled up, it looks like the whole curve.

> Both understandings, modern and ancient, locate the unified source of Nature's perfect order in a single, self-interacting field of intelligence at the foundation of all the Laws of Nature. This field sequentially creates, from within itself, all the diverse Laws of Nature governing life at every level of the manifest universe. [34, p. 78]

This is like the mathematical process of creating a fractal through an iterative process as seen in the next section.

7.2. **Construction of Fractals.** Fractals illustrate how the self-referral dynamics of a simple system can create extraordinary diversity, parallel to the way that pure consciousness, interacting with itself alone, gives rise to the full range of creation as described above.

Creating a fractal is really nothing more than making scaled copies of an original basic shape over and over. The basic shape is like the silent "self" in the process of self-referral and the repetition and scaling are like the dynamic value of creation.

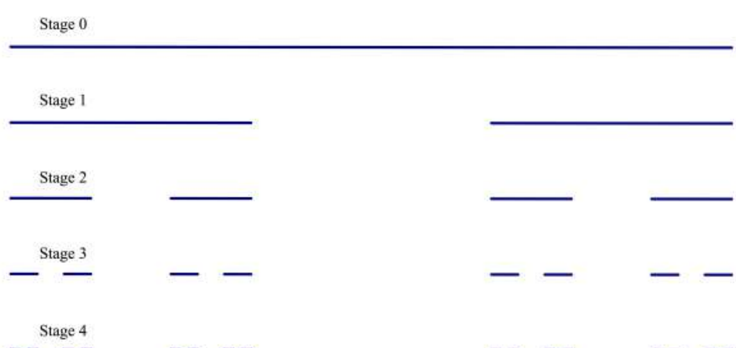

Figure 15. First stages of the Cantor set

To construct the Cantor set, for example, begin with a straight line segment (Stage 0); see Figure 15. Remove the middle third of the segment, leaving the two congruent segments of Stage 1. Each of these two segments is similar to the original segment, scaled down by a factor of 1/3. To obtain Stage 2, remove the middle segment of each of these two segments. Note that in Stage 2, we have four similar copies of Stage 0, but two similar copies of Stage 1. Continue on in this way, removing the middle third of each segment belonging to one stage to get the smaller segments belonging to the next stage. The Cantor set is the set of points that remains after this process has been performed infinitely many times; at the final stage, the Cantor set is similar to a half of itself, a quarter of itself, or an eighth of itself—and so on infinitely.

Georg Cantor (1845–1918), the originator of the Cantor set, discovered many of its surprising properties: it has infinitely many points; it has just as many points as the original segment; it consists only of individual points; it does not contain any segments at all, even though at each successive stage there are more and more segments.

Figure 16. Edge and its replacement in the iterative construction of the Koch snowflake

Another simple fractal is the Koch snowflake. Stage 0 is an equilateral triangle. To get Stage 1, replace the middle third of each edge of the triangle with two sides of an equilateral triangle, as shown in Figure 16. Stage 1 has 12 edges, as shown in Figure 17. At Stage 2, there are 48 edges, and so on. Keep repeating this procedure, replacing each edge with the four edges of Figure 16, scaled to fit the edge that it replaces. The Koch snowflake, Figure 18, is the shape that results as the end stage of infinitely many iterations of this procedure. Like the Cantor set, it is a purely abstract mathematical structure that cannot be drawn on paper.

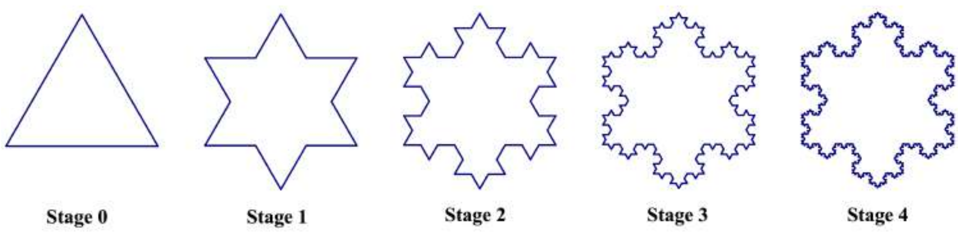

Figure 17. First stages of the Koch snowflake

The construction of these fractals illustrates very clearly the description of the self-referral structure of consciousness given by Maharishi in verse 8 of Chapter 9 of the *Bhagavad-Gita*:

> *Prakṛitiṁ swām avashtabhya visṛijāmi punaḥ punaḥ*
> *Curving back upon My own Nature, I create again and again.*
>
> [35, p. 37]

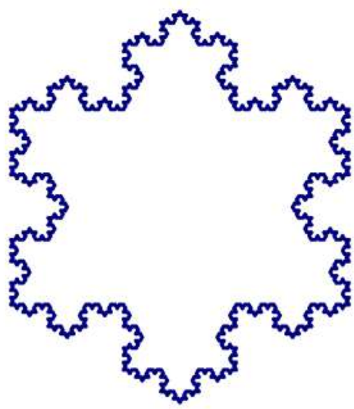

Figure 18. The Koch snowflake

7.3. **Fractals in art.** Artists recognized and imitated the fractal structure of their natural environment even before mathematicians began to study fractals. For example, fractal structures appear in many African designs [20], Celtic illuminated manuscripts [41], and Persian carpets [42]. *The Great Wave off Kanagawa* by Japanese artist Hokusai (1760–1849) shows the fractal structure of an ocean wave. Modern architecture also uses fractal-like structures [56, pp. 325–354, 513–524]. Fractal geometry in art may also be a reflection of the way that artists create their work; Bales [7, 8] proposes that the fractal appearance of certain quilts is the result of the iterative way the quilters work.

8. Non-Euclidean geometry

For thousands of years, there was only one known geometry—the geometry common to the ancient Hindus, the ancient Egyptians, the ancient Greeks, the Mayans, and others [12, 31]. Euclid (c. 330–c. 270 BCE) formalized this geometry in *The Elements* [23], a systematic development of geometry from first principles. Long regarded as the model of presenting knowledge, *The Elements* lists postulates and common notions, the assumptions or rules that guide the development of Euclidean geometry. From those first fundamental assumptions, Euclid derived all of his propositions logically and sequentially. In this way, Euclidean geometry is structured in layers, from the subtlest foundational layer of postulates and common notions through more expressed layers of elementary propositions, to the complex layers that include the Pythagorean theorem and the construction of the five Platonic solids.

With this firm foundation, Euclidean geometry was considered to be the only possible geometry until several major discoveries were made in the nineteenth century, when János Bolyai (1802–60) and Nicolai Lobachevsky (1792–1856), working separately, discovered another kind of geometry [55]. They questioned one of Euclid's postulates, the fifth or parallel postulate, which says that given a line ℓ and a point P not on the line, there exists one and only one line ℓ' through the point

P that is parallel to the original line ℓ, as shown in Figure 19. By changing the parallel postulate—in other words, by operating on the subtlest layer of Euclidean geometry—they were able to create a new geometry, called hyperbolic geometry.

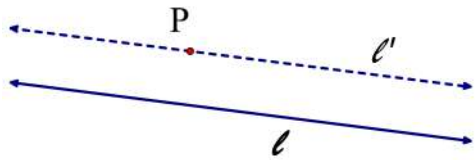

FIGURE 19. The unique line ℓ' through point P that is parallel to the line ℓ .

This new geometry was recognized to be just as valid, just as consistent, and just as true as Euclidean geometry only when, in 1868, Eugenio Beltrami (1835–1900) created a model for hyperbolic geometry, the pseudosphere, from within the structure of Euclidean geometry.

Hyperbolic geometry was joined by another new geometry, elliptic geometry, developed by Bernhard Riemann (1826–66). He showed how the surface of the sphere also satisfied all of Euclid's postulates except the parallel postulate. These new geometries were shown to be of practical as well as theoretical interest when Albert Einstein (1879–1955) used them in his theory of general relativity [48].

Furthermore, Bernard Riemann's study of the sphere showed that elliptic geometry is just as consistent, valid, and true as the other two geometries. Geometry was now not one possibility but a field of possibilities.

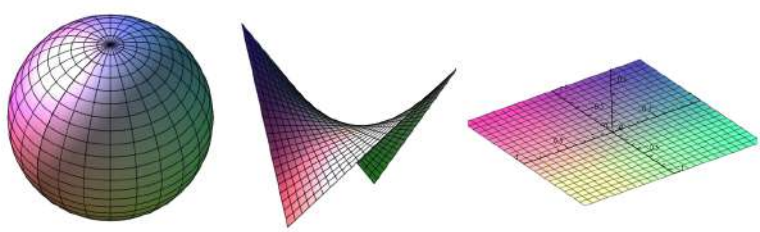

FIGURE 20. Geometries. The surface on the left has Riemannian or elliptic geometry, the surface in the middle is hyperbolic, and the Euclidean plane is on the right.

These geometries look different, as shown in Figure 20. We see the shapes associated with these geometries everywhere around us. The flat surfaces of buildings and man-made objects are examples of Euclidean geometry. Round or spherical objects, like oranges, apples, a tennis ball, and the human head, are examples of elliptic geometry. The surfaces of hyperbolic geometry are like saddles or ruffles and can be seen in the inner curve of the elbow or in kale leaves.

Art students will find these three geometries and their interpretations everywhere in art. The flat surfaces of architecture and man-made items belong to Euclidean geometry. Natural forms like the human body, fruit, flower petals, and so on belong to non-Euclidean geometry. It is not uncommon to see an artist create a striking contrast between the Euclidean geometry of architecture and other man-made forms on the one hand and the non-Euclidean geometry of living forms on the other. *My Parents*[8] by David Hockney (b. 1937) is an example of this.

Artists may depict the surfaces they see realistically, as when Leonardo da Vinci drapes a flat Euclidean cloth over a round elliptic knee, shown in Figure 21. Other artists distort the geometries they see. Pablo Picasso (1881–1973) and the cubists flatten curved surfaces until they become Euclidean; see Picasso's *Portrait of Ambroise Vollard*[9] for an example. Other artists, like Diego Rivera (1886–1957) in *The Flower Carrier*,[10] emphasize the spherical nature of what they see. Kazimir Malevich,[11] Piet Mondrian (1872–1944), and the Minimalists were fascinated by flat surfaces. M.C. Escher based his *Circle Limit*[12] series on the Poincaré disc model of the hyperbolic plane [22, pp. 125–126].

FIGURE 21. Leonardo da Vinci, Study of a Drapery for the Virgin in the *Virgin and Child with St. Anne and a Lamb*, 1503 (Dover)

Through analysis of the types of geometry in a work of art, the viewer is able to see the diversity of life presented by the artist, to see how the artist integrated the diversity into the wholeness of the work, and to more deeply appreciate the skill and intentions of the artist.

[8] http://www.tate.org.uk/art/artworks/hockney-my-parents-t03255

[9] http://www.arts-museum.ru/data/fonds/europe_and_america/j/1001_2000/7199_Portret_Ambruaza_Vollara/index.php?lang=en

[10] https://www.sfmoma.org/artwork/35.4516

[11] See *Untitled*, https://www.guggenheim.org/artwork/2601, for example.

[12] http://www.mcescher.com/gallery/recognition-success/circle-limit-iii/

9. Topology

The subtlest kind of geometry is topology, which uses only the very simple relationships of set theory—membership and inclusion—for its definition. In fact, a topological space is simply a set of points together with certain relationships on the subsets of that set. This level is prior to measurement of length, area, and angle, so topology concerns itself only with properties of the organization of points independent of the measurement of length, area, or angle. In topology, two different shapes are considered to be equivalent or indistinguishable if one can be stretched or twisted into the other without tearing, cutting, or gluing. We could imagine that shapes are made of very stretchy rubber that can be expanded or contracted at will. For this reason, topology is frequently called "rubber sheet geometry." Thus, a square, circle, and triangle are topologically equivalent because any one of the three can be stretched into each of the others.

A famous example used by topologists is that a "donut" and "coffee cup" are topologically equivalent; Figure 22 indicates how a donut-shaped topological space could be transformed without cutting or gluing into a cup-shaped space. An interesting example of a topological space is the Möbius strip, which can be made from a strip of paper that has been twisted by one half-turn and joined, as shown in Figure 23.

Figure 22. A topological transformation of a donut into a coffee cup

Figure 23. Construction of a Möbius band from a strip of paper. The upward-pointing and downward-pointing arrows are matched after rotating an end of the paper through one-half twist.

Many artists are fascinated by the topological transformations that stretch, twist, and bend familiar shapes. El Greco (1541–1614) and Amedeo Modigliani (1884–1920) both elongated the human figure. Salvador Dali (1904–89) stretched and warped objects in paintings such as *The Persistence of Memory* and *The Elephants*. In his *Bathers* paintings, Picasso stretches and bends his subjects almost beyond recognition. Sculptor Alberto Giacometti (1901–66) stretched the human form until it was almost thread-like; Constantin Brâncuşi (1876–1957) created the sculptures *Kiss* and *Bird in Space*, which are quite radical topological transformations of their declared subject matter. Other artists such as M.C. Escher and Max Bill (1908–94), were fascinated by the specific surfaces studied by topologists. Max Bill made versions of the Möbius band in granite, bronze, and concrete. Robert R. Wilson (1914–2000) designed a stainless steel sculpture in the shape of a Möbius band for the FermiLab in Illinois.[13] Keizo Ushio (b. 1951) is a Japanese stone sculptor who effectively uses Möbius bands in conjunction with other shapes [27].

M.C. Escher gives us a few unforeseen lessons in topology. The woodcut *Möbius Strip II (Red Ants)* in Figure 24 has ants crawling on the full length of a lattice-work Möbius strip, showing that a Möbius strip has only one side. *Möbius Strip I* in Figure 25 shows that cutting a Möbius strip down the middle leaves it in one piece.

FIGURE 24. M.C. Escher, *Möbius Strip II (Red Ants)*, 1963, woodcut

FIGURE 25. M.C. Escher, *Möbius Strip 1*, 1961, wood engraving and woodcut

Life is structured in layers. Maharishi explains that the subtlest layers of life are the most powerful. Pure consciousness is the subtlest layer of life and is therefore the most powerful [36, pp. 4–5]. In this section, we have seen that topology is a very subtle layer of geometry, and it should follow that topology is a very powerful branch of mathematics. In fact, topology has applications in some of the subtlest and most powerful areas of science and technology: M-theory in physics, data analysis, quantum computing, and the study of DNA and neural networks in biology.

[13]`https://history.fnal.gov/sculpture.html`

10. Maharishi Science and Technology of Consciousness

In the previous five sections, we have discussed how specific topics in geometry are connected to art and we have seen these connections related to Maharishi Science and Technology of Consciousness. In this section, we will consider several topics in the science of consciousness more broadly and see their connections to geometry and art. For students in the course *Geometry for the Artist*, these themes support their understanding of geometry and the analysis of specific works of art.

10.1. **Consciousness is infinite and unbounded.** The field of pure consciousness is experienced during the practice of the Transcendental Meditation technique to be infinite and unbounded [44, p. 13]. Maharishi describes pure consciousness in this way:

> It is the unlimited vastness of pure existence or pure consciousness, the essential constituent and content of life. It is the field of unlimited, unbounded, eternal life, pure intelligence, pure existence, the Absolute. [36, p. 8]

This is especially relevant to the student of art and mathematics; Maharishi states that the goal of art and the goal of science are both infinity [24, p. 39]. Further, he uses the measure of unboundedness in a work of art as an indication of its value:

> Art is a way of expression that can belong to all of the five senses, as well as to the mind, the intellect, and the ego; art is a way of expression. If that expression indicates the direction of unboundedness, immortality, and bliss, if it inspires those values and indicates those qualities of pure consciousness, then it is to be considered successful art. Through the means of one sense, it takes the viewer to unboundedness, which eventually he sees as his own Self. So the unfoldment of the Self in greater degrees is the purpose of art. [24, pp. 291–292]

Students experience the field of pure consciousness in their meditation and can use this experience of unboundedness as a tool to help them go more deeply into mathematics and art.

The presence of infinity and unboundedness is everywhere in geometry. The Euclidean plane and the lines it contains extend infinitely without boundary. Even a line segment of finite length has infinitely many points. Fractals are defined in terms of an infinite sequence of iterations of a geometric construction. Topology studies the infinite variety of all possible topological spaces.

Many artists have endeavored to give a feeling or sense of the infinite in their work. Students with regular experiences of the infinite are able to more easily resonate with this feeling of infinity in a work of art.

Max Beckmann (1884–1950) made it very clear that his work expresses the infinite, invisible field that lies beyond the finite visible world around us:

> What I want to show in my work is the idea which hides itself behind so-called reality. I am seeking for the bridge which leads from the visible to the invisible, like the famous cabalist who once said: "If

> you wish to get hold of the invisible you must penetrate as deeply as possible into the visible." [29, p. 167]

M.C. Escher in his essay "Approaches to Infinity" brings out the artist's desire to portray the infinite in art, "to penetrate all the way into the deepest infinity right on the plane of a simple piece of drawing paper by means of immovable and visually observable images" [22, p. 123]. He describes his own attempts to represent the infinite using geometric principles and finds that "[t]here is something breathtaking in such laws" [22, p. 124].

10.2. **Creation through a process of self-referral.** In Section 7, we saw that fractals demonstrate the process of creation through self-referral. This process is present everywhere in art—art is the product of the artist's inner life.

Art begins within the self-interacting reverberations of the artist's consciousness referring to itself alone. The interaction of these reverberations with the subjective impulses of the artist's feelings is brought to life on the surface of the canvas.

In this way, art is naturally and inevitably a self-referral process and depends on the self-interacting dynamics of the consciousness of the artist. As Maharishi (cited in [10, p. 332]) puts it:

> The artist comprehends the outlines of the figure—maybe a long face or a short nose—in his consciousness, and then he wants to depict it on marble, on paper, on clay, or on wood; he carves the wood, but he carves the wood to match the picture he contains in his awareness.

The self-portrait is a very concrete example of self-referral in art; the artist gives visual expression to feelings about his or her own self. The process of self-referral is also evident in an individual artist's development, how themes in earlier works of art are developed, refined, and matured in later works.

Besides fractals, there are many other examples of self-referral or self-interaction in mathematics. The symmetry transformations of a symmetric design or pattern give a self-interacting dynamics, showing how some parts of the design or pattern are the same as other parts. The theorems of Euclidean geometry are the result of the self-interacting dynamics of the axioms. Topology depends on the interaction of open sets.

10.3. **The full range of life from silence to dynamism.** The full range of life extends from dynamic activity to deep silence. Outer, relative life is active, ever-changing, and dynamic. Thoughts, feelings, and intuitions of the mind are less active. Underlying these levels is the field of pure consciousness, experienced during the Transcendental Meditation technique as non-active, unchanging, and silent. An artist who wants to capture the full value of life must capture this range into a work of art if the work is to be fulfilling, because, as Maharishi points out: "Extreme dynamic value, extreme silent value, both together make art—make the action waves of bliss, waves of bliss" [24, p. 322].

Artists fully recognize the importance of the presence of both silence and dynamism in art. Kasimir Malevich discusses harmonizing the opposite values of dynamism and silence in *The Non-Objective World*:

> Life wishes not to live but to rest—it strives not for activity but for passivity. For this reason, agreement among the dynamic or static values of the additional element affecting the system is taken for granted, and a "bringing-into-agreement" of the dynamic elements—systematizing them, that is—amounts to transforming them into static elements, for every system is static (even when it is in movement), whereas every construction is dynamic because it is "on the way" toward a system. [38, p. 14]

How skillfully an artist can integrate, harmonize, or "bring into agreement" these opposite values—living and resting, dynamic and static—into the wholeness of a work of art determines how great an impact the work will have on the viewer. Artists do this in different ways. L. Hilberseimer contrasts the work of Kasimir Malevich with that of Piet Mondrian in terms of how each deals with dynamism and silence [38, p. 8]: "Malevich's color concept was static but his concept of form, on the other hand, was dynamic. This stands in sharp contrast to the Neo-Plasticism of Piet Mondrian, in which the forms are static while the colors constitute the dynamic element."

Wassily Kandinsky starts his discussion of the elements of art [32] with recognizing the point as "the proto-element of painting." He views "the geometric point as the ultimate and singular **union of silence and speech**" [32, p. 25], and from that union, he sees the whole of art emerging.

The range of mathematics also is from dynamism to silence. Each area of mathematics has specific kinds of dynamism—transformations or functions—and specific kinds of silence or non-change—the invariants of the transformation. And, as in other areas of life, it is the invariants that give greater power and understanding.

10.4. **Life is structured in layers.** All of the physical world around us is structured in layers, from the galaxies, to the solar system, to our planet, to individual plants and animals, to organ systems, to molecules, atoms, subatomic particles, and finally to the unified field at the basis of the more expressed levels of life. The subjective world of the artist or mathematician is also structured in layers, from the senses, to the mind, to the emotions, to the intellect, to the ego, and finally to the field of pure consciousness or Being, experienced as the Self of an individual. Maharishi describes the qualities of the field of Being in this way:

> Underneath the subtlest layer of all that exists in the relative field is the abstract, absolute field of pure Being which is unmanifested and transcendental. It is neither matter nor energy. It is pure Being, the state of existence.
>
> This state of pure existence underlies all that exists. Everything is the expression of this pure existence or absolute Being which is the essential constituent of all relative life. [36, p. 5]

So, too, a work of art is structured in layers. An effective work of art can lead the viewer's awareness from the superficial, surface level of the work, to deeper levels, to the transcendental level, as Maharishi brings out:

> The nature of life, being expressive, being progressive, unfolds the inner values of life, and this is precisely what art is—the expression of fuller values of life. [24, p. 198]

Deeper levels of life are more powerful, as we can see by comparing power at the molecular level released by burning and power at the atomic level released in a nuclear reactor.

In mathematics, deeper levels are those that are more abstract, those that are more inclusive and more universal. Geometry uncovers patterns and relationships that depend on measurement of length, angle, and area. Topology is more abstract and, as we have seen, more powerful, locating patterns and relationships that are subtler than those of geometry, relationships that are unchanged when a shape is distorted by stretching or shrinking.

An artist must be able to encompass this full range of life subjectively in order to express it in the physical creation of art. When an artist can lead the viewer to this unbounded level of life, the purpose of art is achieved. We see an example of this in *Liberation* by M.C. Escher, Figure 26, which shows free-flying birds evolving from a pattern of triangles. Here, the symmetric tiling of patterns has the simplicity and abstraction of the unbounded field of consciousness, but gives rise to the physical diversity of a flock of birds.

A work of art makes an initial sensory impression on the viewer; this includes the shapes, colors, and content of the picture. If, as some report when viewing art, an experience of wholeness or transcendence occurs, the artist has been able to truly enliven the subtlest, deepest level of the viewer's consciousness.

11. Conclusion

Our exploration of geometry and art has uncovered many deep connections between them and with the science of consciousness. With an understanding of geometry, we can more thoroughly analyze the structure of a work of art and see how the artists' expressions convey what they see in their world. Making connections between the qualities of consciousness and geometry on the one hand to the structure of a work of art on the other hand leads the viewer to a greater appreciation of the wholeness of the work.

Teaching geometry in the context of connections to consciousness and its applications in art makes geometry relevant to students. *Geometry for the Artist* is a popular course at Maharishi University of Management, and students often use what they have learned in this course later in their artwork.

I hope that this paper has been able to demonstrate the effectiveness of the holistic interdisciplinary approach of Consciousness-Based education. I will leave the final words to the students themselves:[14]

- This course was extremely helpful in understanding how math reflects the structure of creation because I got a glimpse at how artists create worlds on a small canvas, and it began to unlock realization of how creation uses the same principles on a much smaller, yet massively larger scale. ... I think math in terms of art (creation) is how many of the great wonders,

[14] All comments from students who took the course in September, 2015, are given in Appendix A.

FIGURE 26. *Liberation* by M.C. Escher, 1955. Lithograph

inventions, and architecture of the Renaissance were created; the artists understood it on a much deeper level.

- *Geometry for the Artist* goes beyond the analysis of two disciplines interacting with each other. In this course we study the most infinite nature of art and of geometry. We use this exploration to find the geometrical nature of infinity in art. And, how that geometry can be seen as the basis for the expression of infinity that art can have.
- Art has always been a dominantly intuitive area for me. I see a work of art and get a sense of it. When I try to intellectualize it too much I find myself without the proper language to describe it. Using both geometry and Consciousness-Based education in this class helped me to integrate intellect and intuition and gave me a language to do so that seems more authentic than contrived.

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Appendix A: Student Comments

Given below are all student responses to the question "Please add any comments you might have about Consciousness-Based Education, particularly with reference to this course" given to the students of *Geometry for the Artist* in September 2015. They have been lightly edited for clarity.

Student A: Very well done in this course of showing the infinite value in shapes and how artists and mathematicians are very much the same in their expression. I like how we always related the lessons to Maharishi Vedic Science. I have a totally different view toward math thanks to your class.

Student B: Consciousness-Based education is the only way to go! It is the only way to feel fulfilled while within a classroom setting. Connecting math to art makes it seem a little more meaningful to those who could care less about math, as people wish to learn things in how it pertains to their own life.

Student C: With a course like this, learning how unbounded consciousness is the source of all art expression and geometry has opened my eyes to the boundless potential for my own self-expression.

Student D: This course was extremely helpful in understanding how math reflects the structure of creation because I got a glimpse at how artists create worlds on a small canvas, and it began to unlock realization of how creation uses the same principles on a much smaller, yet massively larger scale. Math has never seemed more practical than when taught in regards to artistry. Before, any math besides arithmetic and basic algebra seemed useless in my life, but now I see I am surrounded by it. I think math in terms of art (creation) is how many of the great wonders, inventions, and architecture of the Renaissance were created; artists understood it on a much deeper level.

Student E: *Geometry for the Artist* goes beyond the analysis of two disciplines interacting with each other. In this course we study the most infinite nature of art and of geometry. We use this exploration to find the geometrical nature of infinity in art and how geometry can be seen as the basis for the expression of infinity that art can have. In Consciousness-Based education we develop our own infinite nature through Transcendental Meditation. We also go beyond learning objective knowledge and use our subjective experience as an equal tool for gaining knowledge. When I can find myself in the objective knowledge I am gaining an infinite relationship with it.

Student F: Art has always been a dominantly intuitive area for me. I see a work of art and get a sense of it. When I try to intellectualize it too much I find myself without the proper language to describe it. Using both geometry and Consciousness-Based education in this class helped me to integrate intellect and intuition and gave me a language to do so that seems more authentic than contrived. I appreciated that we were encouraged to make these connections in the classroom context instead of making these connections outside of the classroom and not having space to express these things in the classroom. Consciousness-Based education offers a language to describe underlying aspects of consciousness intellectually, and also, through Transcendental Meditation, allows for me to become more sensitive and attuned to these patterns/flavors/aspects of consciousness and how consciousness moves in my own life. This was supportive to me finding authentic and meaningful connection to geometry and geometry as it related to art. In high school, I struggled the most with geometry of all math classes. I relied on rote memorization, and was rewarded for that with an "A" grade, but made no meaningful and lasting connection. This class and the context it was taught in actually facilitated a connection and relationship to the knowledge.

Student G: When education is based in consciousness, it means something. I have never had an easy time in math class before taking *Geometry for the Artist*. However, I have always viewed math as an entity separate from my own Being, and an evil entity at that. Dr. Gorini introduced math to me as an expression of the infinite. Math, like me, is part of the universe in ecstatic motion, so I should try to value it as such. This course has been a breeze and so much fun. Consciousness gives life to knowledge and a sense of purpose to daily activities, including school and homework.

Appendix B: List of Course Topics

Below is a list of the eighteen lessons of the course along with their science of consciousness themes.

Lesson 1: Geometry and Art: From Point to Infinity
Lesson 2: Classifying Symmetric Designs: Locating Nonchange within Change
Lesson 3: Classifying Band Ornaments: Locating Nonchange within Change
Lesson 4: Classifying Tilings: Locating Nonchange within Change
Lesson 5: Symmetry in the Work of Escher: Unbounded Creativity
Lesson 6: Linear Perspective: Connecting Knower and Known
Lesson 7: Checkerboards in Perspective: Pure Knowledge has Organizing Power
Lesson 8: Circles in Perspective: Harmony in Natural Law
Lesson 9: Two-Point and Three-Point Perspective: The Full Range of Creation
Lesson 10: Perspective in the Work of Escher: Expressing Inner Experience
Lesson 11: Similarity and Proportion: Unifying Differences
Lesson 12: Pictorial Composition: Knowledge has Organizing Power
Lesson 13: Fractals: The Part Contains the Whole
Lesson 14: Dynamical Systems and Chaos: Creation through Self-Referral
Lesson 15: The Mandelbrot Set: From Point to Infinity
Lesson 16: Lines, Curves, and Curvature: Creating Dynamism from Silence
Lesson 17: Non-Euclidean Geometries: Consciousness as a Field of All Possibilities
Lesson 18: Topology: Creating from the Home of All the Laws of Nature

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